

## Basics of Counting

### Example

1. How many three digit odd numbers are there with distinct digits?

**Solution:** First we consider the last digit. There are 5 choices(1, 3, 5, 7, 9). Then, there are originally 9 choices for the first digit, but now there are only 8 because one is taken for the last digit. And finally there were originally 10 choices for the second digit, but two have already been chosen so there are 8 left. Therefore, there are a total of  $5 \cdot 8 \cdot 8 = 320$  numbers.

### Problems

2. True **FALSE** When dealing with a multi-stage counting problem, it doesn't matter in which order we consider the stages.

**Solution:** As seen in the example, we had to consider the last digit, then the first, then the second. If we dealt with the second and then the first, then we would have to break up the problem into cases of whether the second was 0 or not.

3. True **FALSE** The product rule and the sum rule cannot be used together.

**Solution:** Sometimes you have to use both rules.

4. How many three letter initials with distinct letters can people have?

**Solution:** There are 26 choices for the first letter, 25 for the second, and 24 for the third. So  $26 \cdot 25 \cdot 24$ .

5. How many ways are there to line up 5 different people?

**Solution:** There are 5 choices for the first spot, 4 for the second and etc. so  $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$ .

6. How many ways are there to choose a single representative of the math department if there are 30 faculty members and 60 students in the department (the representative can be a faculty member or a student).

**Solution:** The representative can either be a faculty member or a student which suggests adding, so  $30 + 60 = 90$  ways.

7. How many different Minneapolis, MN phone numbers are there if the area code has to be (612), (763), or (952), and the remaining 7 digits can be whatever?

**Solution:** There are 3 ways to choose the area code and then for each of the other 7 digits, there are 10 choices so  $3 \cdot 10^7$ .

8. A palindrome is a number (or phrase) that is the same if you reverse it. How many 3 digit palindromes are there?

**Solution:** There are 9 choices for the first digit, 10 for the second, and the third has to be the same as the first so  $9 \cdot 10 = 90$  different palindromes.

9. How many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if the bride must be next to the groom? If the bride cannot be next to the groom?

**Solution:** If the bride is next to the groom, there are 2 ways to place them (bride on left or right). Then we can consider them one item and arrange them with the remaining 4 people which can happen in  $5!$  different ways. So the answer is  $2 \cdot 5! = 240$ .  
The bride can either be next to the groom or not next to the groom. Therefore, summing these two will give all possible ways to arrange them or  $6!$ . Therefore, there are  $6! - 240 = 480$  different ways to position the bride not next to the groom.